1. Parallelization with MPI (300 Points)

Parallelize the Jacobi method of the partdiff program with MPI. The results shall remain the same when compared to the serial version.

There are two cases to think about:

1. Jacobi method with termination after iterations
2. Jacobi method with termination after precision

Make sure that the results with multiple processes are identical to the serial program version. If that is not the case, your program is wrong!

Requirements and Hints

- The program must not be slower than the serial version.
- At no point is one process allowed to store the complete matrix in its memory.
- The program still has to work with one process.
- The program must work with an arbitrary amount of processes.
- Create a separate function for the MPI-parallelized Jacobi method.

Hint: Use the displayMatrixMpi function that is provided in the materials for printing out the distributed matrix. Also take a look at its comments to understand how the memory layout for each process is supposed to look.

Hint: It is hard to gauge the time that this task will take, because it heavily relies on your previous knowledge and also luck that your first try will yield a working MPI implementation. Debugging complex errors for this task can take a lot of time, therefore start early.
2. Performance Analysis (120 Points)

Measure the performance of your program and visualize the runtime for the following configurations in an appropriate diagram. A configuration \((N, P, I)\) in the following means that your program will run on \(N\) nodes with a total of \(P\) evenly distributed processes and \(I\) interlines.

\[(1, 1, 836), (1, 2, 1,182), (1, 3, 1,448), (1, 6, 2,048), (1, 12, 2,896), (1, 24, 4,096), (2, 48, 5,793), (4, 96, 8,192), (8, 192, 11,585)\]

Also compare your program to the original serial version of the program. The shortest run should take at least 30 seconds; choose appropriate parameters!

Repeat each measurement at least three times to get sensible averages. For this, you can use the `hyperfine` tool which can be loaded with `module load hyperfine`.

For \(N\) nodes, \(P\) processes, \(n\) iterations and \(I\) iterations you can use the following script:

```
#!/bin/bash
#SBATCH --nodes=N
#SBATCH --ntasks=P
#SBATCH --exclusive
#SBATCH --partition=vl-parcio

srun --mpi=pmi2 ./partdiff 1 2 I 2 2 n
```

The measurements should be done with SLURM on the compute nodes of the cluster. Document the hardware specifications used for the measurements (processor, core count, available main memory, etc.).

In the materials, you can find prepared job scripts.

Visualize all results in appropriately labeled diagrams. Write about a quarter page of interpretation for these results.

3. Hybrid Parallelization (120 Bonus Points)

Also parallelize your MPI version of the Jacobi method with OpenMP.

Measure the performance of your hybrid program and compare the runtimes of the following configurations in a diagram:

- 1 process \(\times\) 24 threads
- 2 processes \(\times\) 12 threads
- 3 processes \(\times\) 8 threads
- 6 processes \(\times\) 4 threads
- 12 processes \(\times\) 2 threads
- 24 processes \(\times\) 1 thread
Use 4,096 interlines for this. The shortest run should take at least 30 seconds; choose appropriate parameters!

Repeat each measurement at least three times to get sensible averages.

The measurements should be done with SLURM on one of the compute nodes of the cluster. Document the hardware specifications used for the measurements (processor, core count, available main memory, etc.) and use the same compute node for each measurement.

Visualize all results in appropriately labeled diagrams. Write about a quarter page of interpretation for these results.

4. Gauß-Seidel Parallelization (300 Bonus Points)

If you are confident in your Jacobi parallelization and want to challenge yourself, try to also parallelize the Gauß-Seidel method.

For termination after iterations, the program shall still yield the exact same results as the serial version. For termination after precision, the program does not have to stop after the exact same number of iterations as the serial version. However, the results must still be the correct ones for the number of iterations that have been performed.

Create a separate function for the Gauß-Seidel method and make sure that this task does not break your Jacobi implementation. The same requirements as for the Jacobi version apply.

Submission

We will count your last commit on the main branch of your repository before the exercise deadline as your submission. In the root directory of the repository, we expect a PP-2024-Exercise-07-Materials directory with the following contents:

- A file group.txt with your group members (one per line) in the following format:

  Erika Musterfrau <erika.musterfrau@example.com>
  Max Mustermann <max.mustermann@example.com>

- The modified Code of the partdiff program in the pde directory (Tasks 1, 3 and 4)
  - Optional: A partdiff-hybrid target in the Makefile which creates a partdiff-hybrid binary with the hybrid parallelization

- A performance-analysis.pdf document with the measured runtimes and your interpretations (Tasks 2 and 3)